Mathematical Modeling of Fish Detection with a Scanning Airborne Laser

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Abstract

MURPHREE et al.[1] previously assessed the feasibility of an airborne remote sensing laser system for the detection of fish with a pulsed laser beam incident normal to the ocean surface. A mathematical model was developed and solved which incorporated the physical interactions involved in the process of laser transmission through the air/sea interface and ocean environment. The results from the developed mathematical model revealed that for laser radiation incident normal to the ocean surface, the power received at an airborne detector from the fish-reflected laser radiation and the signal-to-noise ratio (S/N) are of sufficient magnitude to locate the fish schools with an airborne remote sensing laser system. The assumption of normal incidence in the previous work greatly simplified the mathematical model.[1] However, for a practical operational airborne laser system for fish detection, the laser beam must be swept transverse to the aircraft's flight direction to provide for maximum coverage area. To mathematically model such an operational system, the governing equations must allow for laser radiation incident on the surface at oblique angles. In this analysis, the power reflected from fish and received at the airborne detector is modeled for laser radiation obliquely incident on the ocean surface, as would occur in an operational airborne laser fish detection system. A laser swath width of 75 m transverse to the flight direction was chosen to provide for adequate coverage of the search area in a practical time.

Contents

A mathematical model is developed for the fish-reflected laser power received at the airborne detector as a function of laser power, wind speed and direction, beam radius, angle of incidence of the beam with respect to the surface normal, fish depth, and aircraft altitude. In general, the development of Swennen is followed for determining the time-average underwater laser power probability distribution for a beam of collimated radiation incident at an arbitrary angle to the ocean surface.[2] The statistical distribution of slopes on the ocean surface is obtained from the empirical relationship of Cox and Munk, which uses a Gaussian distribution modified by a Gram-Charlier series to include the effects of wind direction and velocity.[3] The power reflected from the submerged fish and detected at the airborne receiver is determined using the principle of electromagnetic reciprocity.[4]

The power density probability distribution at a point (μ_0,ν_0,z) beneath the ocean surface due to laser radiation incident at an angle θ with respect to the surface normal is given by[2]

$$P_d(\mu_0,\nu_0,z,\theta) = \int_\nu \int_\mu P_0 \exp(-\gamma\,|z|\,s\mu)\,\tau \tan\beta s^2\beta \cos\omega_i s\omega_r$$
$$\times(\cos\mu\cos\mu_0 + \sin\mu\sin\mu_0\cos|\nu-\nu_0|)\,p(\alpha,\beta)\,J^{-1}\,\mathrm{d}\mu\,\mathrm{d}\nu \qquad (1)$$

which is the integral over the laser beam illuminated ocean surface facets refracting the laser light to the point (μ_0,ν_0,z). In Eq. (1), P_0 is the laser power density at the ocean surface, τ is the surface transmittance, γ is the volume attentuation coefficient equal to the sum of the volume absorption coefficient and the total volume scattering coefficient, β the angle formed by the line of steepest ascent of the elementary surface and its projection onto the horizontal plane, α the angle formed by the foregoing projection and the plane of incidence ($\alpha=0$ is directed toward the laser source), ω_i the angle of incidence, and ω_r the angle of refraction. Expressions for τ, ω_i, and ω_r are given in Ref. 1. The probability distribution of slopes on the ocean surface, $p(\alpha,\beta)$, is given in Ref. 3, and the Jacobian of the transformation between the variables (μ,ν) and (α,β), J^{-1}, is given in Ref. 2.

In the process of developing the described mathematical model, an error in Ref. 2 in the derivation of the limits of μ was detected. In Ref. 2, a coordinate pair (X_6,Y_6) is used in the computation of the upper limit of μ inside the beam. This pair is the result of the solution of a set of simultaneous equations relating to the problem geometry, and results in an equation for X_6 containing a $\pm\sqrt{\ }$. The sign was chosen positive in Ref. 2, as its selection was stated to be arbitrary. In our analysis of this problem, however, it was noted that the positive sign is valid only when $\nu<\pi$. For the cases $\nu>\pi$, the sign must be chosen negative.

The equation for the fish-reflected laser power received at the airborne detector from fish located at (μ_0,ν_0,z) is determined in Ref. 1 using the principle of electromagnetic reciprocity. The backup paper to this synoptic describes the summation performed over all laser-illuminated underwater points (μ_0,ν_0,z) for a given θ to obtain the total power received at the airborne detector.

Based on the principle of electromagnetic reciprocity, the power received at the airborne detector from fish within a grid element ΔA located at (μ_0,ν_0,z) for a given θ is given by[1]

$$P^{\mathrm{rec}}_{(\mu_0,\nu_0,z,\theta)} = \frac{R_f \epsilon \Delta A A_r}{2\pi^2(h\cos\theta)^2 ab}\left[\frac{P_{d(\mu_0,\nu_0,z,\theta)}}{P_0}\right]^2 p^{\mathrm{rad}} \qquad (2)$$

where R_f is the fish reflectance, h the aircraft altitude, a the semiminor axis and b the semimajor axis of the elliptical

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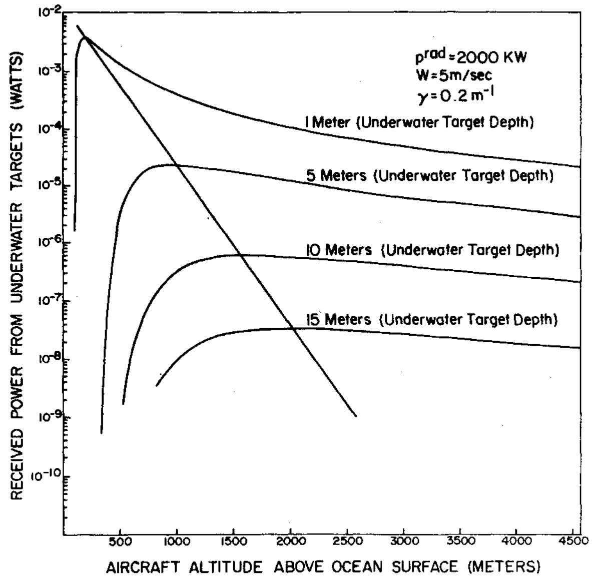

Fig. 1 Minimum received power at airborne detector for 75-m swath width as a function of fish depth and aircraft altitudes.

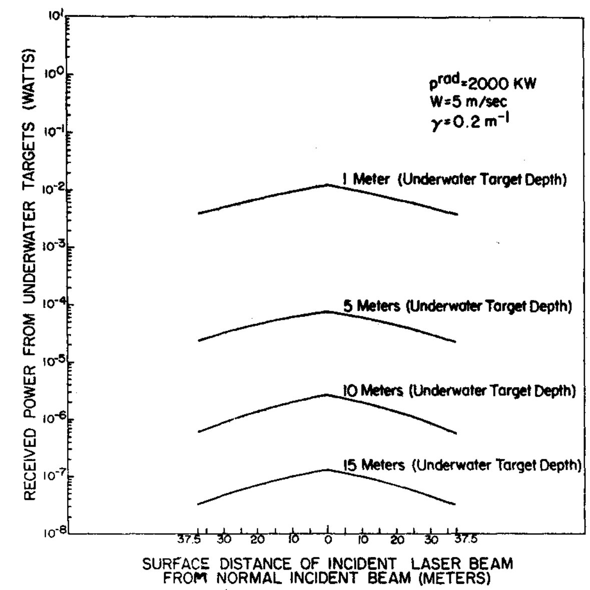

Fig. 2 Received power at the airborne detector for laser beam sweeping across the 75-m swath width for optimum aircraft altitudes.

illuminated surface area, respectively, A_r the receiver area, and ϵ the ratio of the area ΔA_f covered by fish in the grid element of area ΔA located at (μ_0, ν_0, z) to the grid element area ΔA. Table 1 presents the input data for the presented calculations.

The variation of the underwater power density at different grid elements of the underwater coordinate system is presented in the backup paper for three different angles of incidence of the laser beam of 0, 5, and 10 deg. The backup paper also presents an example of the underwater power density at different grid points as a function of wind velocity.

The power received at the airborne detector for the laser beam incident at the extremity of the swath width, i.e., 37.5 m from the normal projection of the aircraft to the ocean surface, is presented in Fig. 1 for different underwater fish depths as a function of aircraft altitude. For a given depth and altitude, this represents the minimum power that would be received at the aircraft during the laser traversal of the 75-m swath width.

Table 1 Input data

Pulsed laser power p^{rad}	2000 kW
Beam radius at water surface a	0.5 m
Grid element area ΔA	0.0314 m^2
Ratio of target area to grid area ϵ	0.5
Fish reflectivity R_f	0.05
Receiver area A_r	0.13 m^2

For low altitudes, the increasing reflectance of the laser beam on the ocean surface with increasing sweeping angle reduces the laser power penetration to the underwater targets, reducing the underwater laser power density. With increasing altitude, the angle of incidence of the laser beam is reduced across the swath width, thereby reducing the amount of surface-reflected laser power; however, the $1/r^2$ dependence of the power received at the airborne detector becomes significant. This is shown in Fig. 1.

An optimum altitude for maximum received power at the airborne detector, therefore, exists for a given underwater target depth. The straight line in Fig. 1 gives the optimum aircraft altitude for maximum received power for a given fish depth.

Using the optimum altitude for maximum received laser power at the airborne detector for a given underwater target depth as determined from Fig. 1, the power received at the airborne detector across the 75-m swath width is presented in Fig. 2. Four different underwater target depths are shown. Figure 2 shows that, while the maximum received power occurs for normal incidence, there is not a large decrease in received laser power with increasing angle of incidence during sweeping of the laser beam across the 75-m swath width for the optimum altitude conditions.

The equipment requirements of an airborne laser system for fish detection and an analysis of the signal-to-noise ratio (S/N) are presented in Ref. 1. The S/N analysis of Ref. 1 is applicable for laser radiation obliquely incident on the ocean surface. A conservative estimate of the S/N range is

$$3.5 \leq S/N \lesssim 12 \tag{3}$$

The results of the developed mathematical model of fish-reflected incident laser radiation from a laser beam sweeping transverse to the aircraft flight direction reveal that the fish-reflected laser power received at the airborne detector, the zone of detectibility (both depth beneath ocean surface and swath width), and the S/N are such that development of an operational scanning airborne laser fish detection system is indeed feasible.

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